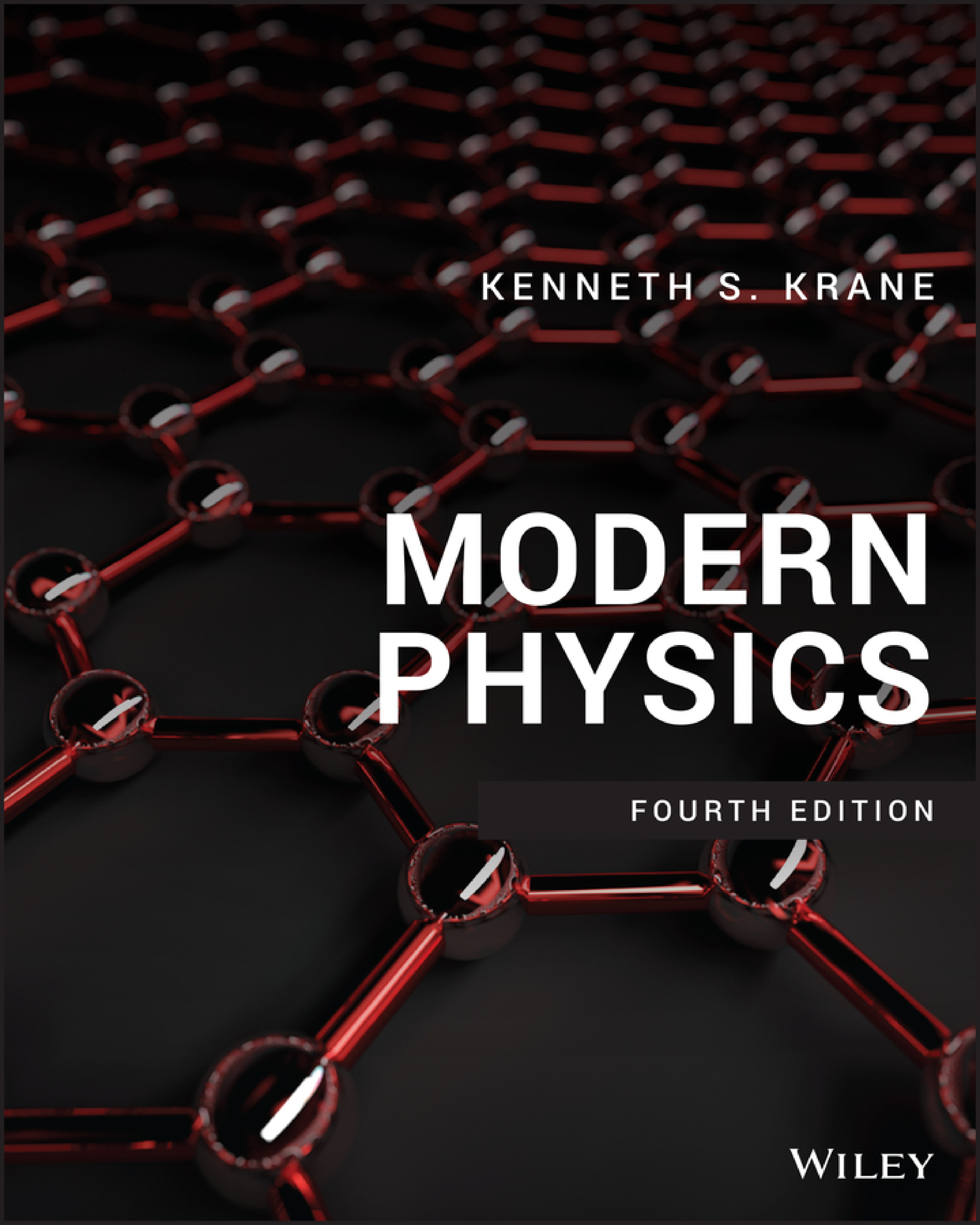

KENNETH S. KRANE

MODERN PHYSICS

FOURTH EDITION

WILEY

MODERN PHYSICS

MODERN PHYSICS

Fourth edition

Kenneth S. Krane
DEPARTMENT OF PHYSICS
OREGON STATE UNIVERSITY

WILEY

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PREFACE

This textbook is meant to serve a first course in modern physics, including relativity, quantum mechanics, and their applications. Such a course often follows the standard introductory course in calculus-based classical physics. The course addresses two different audiences: (1) Physics majors, who will later take a more rigorous course in quantum mechanics, find an introductory modern course helpful in providing background for the rigors of their imminent coursework in classical mechanics, thermodynamics, and electromagnetism. (2) Nonmajors, who may take no additional physics class, find an increasing need for concepts from modern physics in their disciplines—a classical introductory course is not sufficient background for chemists, computer scientists, nuclear and electrical engineers, or molecular biologists.

Necessary prerequisites for undertaking the text include any standard calculus-based course covering mechanics, electromagnetism, thermal physics, and optics. Calculus is used extensively, but no previous knowledge of differential equations, complex variables, or partial derivatives is assumed (although some familiarity with these topics would be helpful).

Chapters 1–8 constitute the core of the text. They cover special relativity and quantum theory through atomic structure. At that point the reader may continue with Chapters 9–11 (molecules, quantum statistics, and solids) or branch to Chapters 12–14 (nuclei and particles). The final chapter covers cosmology and can be considered the capstone of modern physics as it brings together topics from relativity (special and general) as well as from nearly all of the previous material covered in the text.

The unifying theme of the text is the empirical basis of modern physics. Experimental tests of derived properties are discussed throughout. These include the latest tests of special and general relativity as well as studies of wave-particle duality for photons and material particles. Applications of basic phenomena are extensively presented, and data from the literature are used not only to illustrate those phenomena but to offer insights into how "real" physics is done. Students using the text have the opportunity to study how laboratory results and the analysis based on quantum theory go hand-in-hand to illuminate diverse topics such as Bose–Einstein condensation, heat capacities of solids, paramagnetism, the cosmic microwave background radiation, X-ray spectra, dilute mixtures of ^{3}He in ^{4}He, and molecular spectroscopy of the interstellar medium. New or revised discussions of phenomena added to the present edition include the following:

- Particle wave duality and the delayed choice experiment
- The structure and properties of graphene
- Energy levels of quarkonium
- Gravity waves and LIGO
- The Higgs boson
- Test of the relativistic Doppler effect with trapped atoms

- Stellar fusion and the Josephson junction as examples of barrier penetration
- The Franck–Hertz experiment with neon and argon
- The atomic structure and expected properties of elements up to atomic number 118

The 4th edition continues to emphasize and facilitate good teaching practices as revealed in recent physics education research (PER). One of the major themes that have emerged from PER is that students can often learn successful algorithms for solving problems while lacking a fundamental understanding of the underlying concepts. Many approaches to addressing this deficiency are based on preclass conceptual exercises and in-class individual or group activities that help students to reason through diverse problems that cannot be resolved by plugging numbers into an equation. It is absolutely essential to devote class time to these conceptual exercises and to follow through with exam questions that require similar analysis and articulation of the conceptual reasoning.

To illustrate successful techniques for attacking conceptual problems, in this edition, solved conceptual exercises have been added to the text, an average of about 2 per chapter, so that the chapters now include not only many solved numerical problems but also solved conceptual challenges. More details regarding the application of PER to the teaching of modern physics, including references to articles from the PER literature, are included in the Instructor's Manual for this text. The Instructor's Manual also includes more examples of conceptual questions for in-class discussion or exams that have been developed and class tested through the support of a Course, Curriculum and Laboratory Improvement grant from the National Science Foundation.

More than 75 new end-of-chapter problems have been added to this edition. Some of these are confidence-building exercises, but many use actual situations from physics research and present real or simulated data to analyze. Among the topics covered by these new problems are as follows:

- Solar sails
- Inverse Compton scattering
- Franck–Hertz experiment with argon
- Humphreys series in hydrogen atoms
- Rydberg atoms
- X-ray fine structure
- X-ray energy-level diagrams
- Sigma and pi bonds in molecules
- Rotation–vibration structure in CO
- The Wieman–Cornell BEC experiment
- Alpha decay of "stable" Bi-209
- Double beta decay
- Decay of fully ionized Fe-52
- Production of superheavy elements
- Collisions in the LHC
- Hawking radiation

I am grateful to users of the 3rd edition who have offered helpful suggestions and guidance for the preparation of the 4th edition:

- Thomas Greenlee: Bethel University, Saint Paul Campus
- Dawn Hollenbeck: Rochester Institute of Technology

- Denis Leahy: University of Calgary
- Jiali Li: University of Arkansas, Fayetteville
- Andreas Piepke: University of Alabama, Tuscaloosa
- Jean Quashnock: Carthage College

I am also grateful to the many anonymous comments from students who used the manuscript at the test sites. I am indebted to all the reviewers and users for their contributions to the project.

Funding for the development and testing of the supplemental exercises in the Instructor's Manual was provided through a grant from the National Science Foundation. I am pleased to acknowledge their support. Two former graduate students at Oregon State University helped to test and implement the curricular reforms: K. C. Walsh and Pornrat Wattasinawich. I appreciate their assistance in this project.

In my research and other professional activities, I occasionally meet physicists who used earlier editions of this text when they were students. Some report that their first exposure to modern physics kindled the spark that led them to careers in physics. For many students, this course offers their first insights into what physicists really do and what is exciting, perplexing, and challenging about our profession. I hope students who use this new edition will continue to find those inspirations.

Corvallis, Oregon
January 2019

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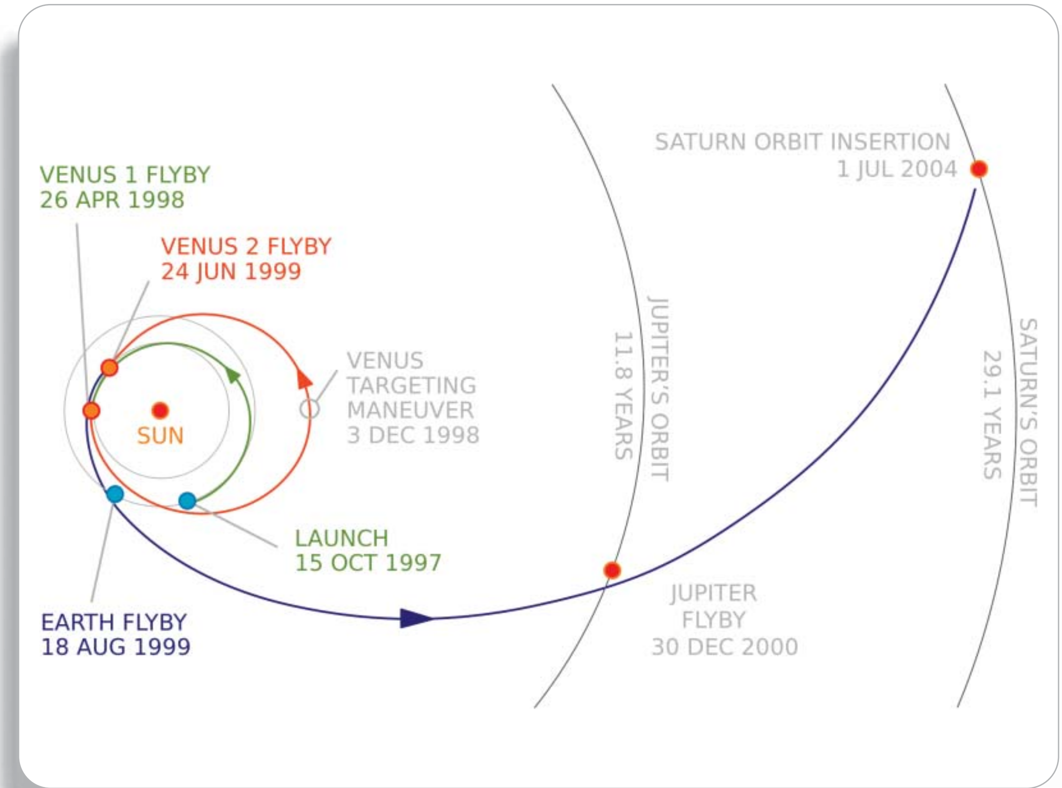
CONTENTS

Chapter 1

SOME DEFICIENCIES OF CLASSICAL PHYSICS



Classical physics, as postulated by Newton, has enabled us to send space probes on trajectories involving many complicated maneuvers, such as the Cassini mission to Saturn, which was launched in 1997 and gained speed for its trip to Saturn by performing four "gravity-assist" flybys of Venus (twice), Earth, and Jupiter. The spacecraft arrived at Saturn in 2004 and ended its mission in 2017 with a plunge into Saturn's atmosphere. Planning and executing such interplanetary voyages are great triumphs for Newtonian physics, but when objects move at speeds close to the speed of light or when we examine matter on the atomic or subatomic scale, Newtonian mechanics is not adequate to explain our observations, as we discuss in this chapter. © NASA

If you were a physicist living at the end of the 19th century, you probably would have been pleased with the progress that physics had made in understanding the laws that govern the processes of nature. Newton's laws of mechanics, including gravitation, had been carefully tested, and their success had provided a framework for understanding the interactions among objects. Electricity and magnetism had been unified by Maxwell's theoretical work, and the electromagnetic waves predicted by Maxwell's equations had been discovered and investigated in the experiments conducted by Hertz. The laws of thermodynamics and kinetic theory had been particularly successful in providing a unified explanation of a wide variety of phenomena involving heat and temperature. These three successful theories—mechanics, electromagnetism, and thermodynamics—form the basis for what we call "classical physics."

Beyond your 19th-century physics laboratory, the world was undergoing rapid changes. The Industrial Revolution demanded laborers for the factories and accelerated the transition from a rural and agrarian to an urban society. These workers formed the core of an emerging middle class and a new economic order. The political world was changing, too—the rising tide of militarism, the forces of nationalism and revolution, and the gathering strength of Marxism would soon upset established governments. The fine arts were similarly in the middle of revolutionary change, as new ideas began to dominate the fields of painting, sculpture, and music. The understanding of even the very fundamental aspects of human behavior was subject to serious and critical modification by the Freudian psychologists.

In the world of physics, too, there were undercurrents that would soon cause revolutionary changes. Even though the overwhelming majority of experimental evidence agreed with classical physics, several experiments gave results that were not explainable in terms of the otherwise successful classical theories. Classical electromagnetic theory suggested that a medium is needed to propagate electromagnetic waves, but precise experiments failed to detect this medium. Experiments to study the emission of electromagnetic waves by hot, glowing objects gave results that could not be explained by the classical theories of thermodynamics and electromagnetism. Experiments on the emission of electrons from surfaces illuminated with light also could not be understood using classical theories.

These few experiments may not seem significant, especially when viewed against the background of the many successful and well-understood experiments of the 19th century. However, these experiments were to have a profound and lasting effect not only on the world of physics, but also on all of science, on the political structure of our world, and on the way we view ourselves and our place in the universe. Within the short span of two decades between 1905 and 1925, the shortcomings of classical physics would lead to the special and general theories of relativity and the quantum theory.

The designation *modern physics* usually refers to the developments that began in about 1900 and led to the relativity and quantum theories, including the applications of those theories to understanding the atom, the atomic nucleus and the particles of which it is composed, collections of atoms in molecules and solids, and, on a cosmic scale, the origin and evolution of the universe. Our discussion of modern physics in this text touches on each of these areas.

We begin our study in this chapter with a brief review of some important principles of classical physics, and we discuss some situations in which classical physics offers either inadequate or incorrect conclusions. These situations are not necessarily those that originally gave rise to the relativity and quantum theories, but they do help us understand why classical physics fails to give us a complete picture of nature.

1.1 REVIEW OF CLASSICAL PHYSICS

Although there are many areas in which modern physics differs radically from classical physics, we frequently find the need to refer to concepts of classical physics. Here is a brief review of some of the concepts of classical physics that we may need.

Mechanics

A particle of mass m moving with velocity v has a *kinetic energy* defined by

$$K = \frac{1}{2} mv^2 \tag{1.1}$$

and a *linear momentum* $\vec{\mathbf{p}}$ defined by

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} \tag{1.2}$$

In terms of the linear momentum, the kinetic energy can be written as

$$K = \frac{p^2}{2m} \tag{1.3}$$

When one particle collides with another, we analyze the collision by applying two fundamental conservation laws:

I. **Conservation of Energy.** The total energy of an isolated system (on which no net external force acts) remains constant. In the case of a collision between particles, this means that the total energy of the particles *before* the collision is equal to the total energy of the particles *after* the collision.

II. **Conservation of Linear Momentum.** The total linear momentum of an isolated system remains constant. For the collision, the total linear momentum of the particles *before* the collision is equal to the total linear momentum of the particles *after* the collision. Because linear momentum is a vector, application of this law usually gives us two equations, one for the x components and another for the y components.

These two conservation laws are of the most basic importance to understanding and analyzing a wide variety of problems in classical physics. Problems 1–4 and 11–14 at the end of this chapter review the use of these laws.

The importance of these conservation laws is both so great and so fundamental that, even though in Chapter 2 we learn that the special theory of relativity modifies Eqs. 1.1–1.3, the laws of conservation of energy and linear momentum remain valid.

Example 1.1

A helium atom ($m = 6.6465 \times 10^{-27}$ kg) moving at a speed of $v_{He} = 1.518 \times 10^6$ m/s collides with an atom of nitrogen ($m = 2.3253 \times 10^{-26}$ kg) at rest. After the collision, the helium atom is found to be moving with a velocity of $v'_{He} = 1.199 \times 10^6$ m/s at an angle of $\theta_{He} = 78.75°$ relative to the direction of the original motion of the helium atom. (*a*) Find the velocity (magnitude and direction) of the nitrogen atom after the collision. (*b*) Compare the kinetic energy before the collision with the total kinetic energy of the atoms after the collision.

Solution

(*a*) The law of conservation of momentum for this collision can be written in vector form as $\vec{\mathbf{p}}_{initial} = \vec{\mathbf{p}}_{final}$, which is equivalent to

$$p_{x,initial} = p_{x,final} \quad \text{and} \quad p_{y,initial} = p_{y,final}$$

The collision is shown in Figure 1.1. The initial values of the total momentum are, choosing the x axis to be the direction of the initial motion of the helium atom,

$$p_{x,initial} = m_{He} v_{He} \quad \text{and} \quad p_{y,initial} = 0$$

The final total momentum can be written as

$$p_{x,final} = m_{He} v'_{He} \cos\theta_{He} + m_N v'_N \cos\theta_N$$
$$p_{y,final} = m_{He} v'_{He} \sin\theta_{He} + m_N v'_N \sin\theta_N$$

The expression for $p_{y,final}$ is written in general form with a + sign even though we expect that θ_{He} and θ_N are on opposite sides of the x axis. If the equation is written in this way, θ_N will come out to be negative. The law of conservation of momentum gives, for the x components, $m_{He} v_{He} = m_{He} v'_{He} \cos\theta_{He} + m_N v'_N \cos\theta_N$, and for the y components, $0 = m_{He} v'_{He} \sin\theta_{He} + m_N v'_N \sin\theta_N$. Solving for the unknown terms, we find

$$\begin{aligned} v'_N \cos\theta_N &= \frac{m_{He}(v_{He} - v'_{He}\cos\theta_{He})}{m_N} \\ &= \{(6.6465 \times 10^{-27}\ \text{kg})[1.518 \times 10^6\ \text{m/s} \\ &\quad - (1.199 \times 10^6\ \text{m/s})(\cos 78.75°)]\} \\ &\quad \times (2.3253 \times 10^{-26}\ \text{kg})^{-1} \\ &= 3.6704 \times 10^5\ \text{m/s} \end{aligned}$$

$$\begin{aligned} v'_N \sin\theta_N &= -\frac{m_{He} v'_{He} \sin\theta_{He}}{m_N} \\ &= -(6.6465 \times 10^{-27}\ \text{kg})(1.199 \times 10^6\ \text{m/s}) \\ &\quad \times (\sin 78.75°)(2.3253 \times 10^{-26}\ \text{kg})^{-1} \\ &= -3.3613 \times 10^5\ \text{m/s} \end{aligned}$$

We can now solve for v'_N and θ_N:

$$\begin{aligned} v'_N &= \sqrt{(v'_N \sin\theta_N)^2 + (v'_N \cos\theta_N)^2} \\ &= \sqrt{(-3.3613 \times 10^5\ \text{m/s})^2 + (3.6704 \times 10^5\ \text{m/s})^2} \\ &= 4.977 \times 10^5\ \text{m/s} \end{aligned}$$

$$\begin{aligned} \theta_N &= \tan^{-1}\frac{v'_N \sin\theta_N}{v'_N \cos\theta_N} \\ &= \tan^{-1}\left(\frac{-3.3613 \times 10^5\ \text{m/s}}{3.6704 \times 10^5\ \text{m/s}}\right) = -42.48° \end{aligned}$$

(*b*) The initial kinetic energy is

$$\begin{aligned} K_{initial} &= \tfrac{1}{2} m_{He} v_{He}^2 \\ &= \tfrac{1}{2}(6.6465 \times 10^{-27}\ \text{kg})(1.518 \times 10^6\ \text{m/s})^2 \\ &= 7.658 \times 10^{-15}\ \text{J} \end{aligned}$$

and the total final kinetic energy is

$$\begin{aligned} K_{final} &= \tfrac{1}{2} m_{He} v'^2_{He} + \tfrac{1}{2} m_N v'^2_N \\ &= \tfrac{1}{2}(6.6465 \times 10^{-27}\ \text{kg})(1.199 \times 10^6\ \text{m/s})^2 \\ &\quad + \tfrac{1}{2}(2.3253 \times 10^{-26}\ \text{kg})(4.977 \times 10^5\ \text{m/s})^2 \\ &= 7.658 \times 10^{-15}\ \text{J} \end{aligned}$$

Note that the initial and final kinetic energies are equal. This is the characteristic of an *elastic* collision, in which no energy is lost to, for example, internal excitation of the particles.

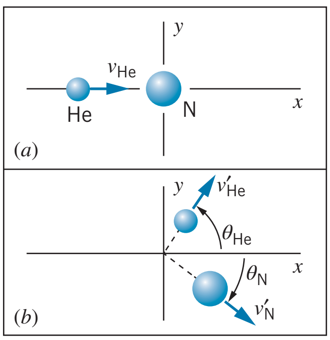


FIGURE 1.1 Example 1.1. (*a*) Before collision; (*b*) after collision.

Example 1.2

An atom of uranium ($m = 3.9529 \times 10^{-25}$ kg) at rest decays spontaneously into an atom of helium ($m = 6.6465 \times 10^{-27}$ kg) and an atom of thorium ($m = 3.8864 \times 10^{-25}$ kg). The helium atom is observed to move in the positive x direction with a velocity of 1.423×10^7 m/s (Figure 1.2). (*a*) Find the velocity (magnitude and direction) of the thorium atom. (*b*) Find the total kinetic energy of the two atoms after the decay.

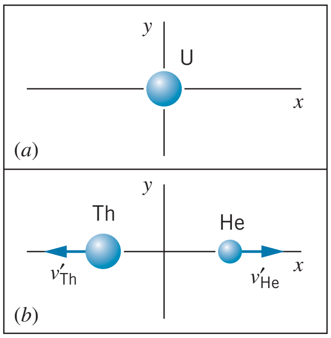


FIGURE 1.2 Example 1.2. (*a*) Before decay; (*b*) after decay.

Solution

(*a*) Here we again use the law of conservation of momentum. The initial momentum before the decay is zero, so the total momentum of the two atoms after the decay must also be zero:

$$p_{x,\text{initial}} = 0 \qquad p_{x,\text{final}} = m_{He}v'_{He} + m_{Th}v'_{Th}$$

Setting $p_{x,\text{initial}} = p_{x,\text{final}}$ and solving for v'_{Th}, we obtain

$$\begin{aligned} v'_{Th} &= -\frac{m_{He}v'_{He}}{m_{Th}} \\ &= -\frac{(6.6465 \times 10^{-27}\ \text{kg})\,(1.423 \times 10^7\ \text{m/s})}{3.8864 \times 10^{-25}\ \text{kg}} \\ &= -2.432 \times 10^5\ \text{m/s} \end{aligned}$$

The thorium atom moves in the negative x direction.

(*b*) The total kinetic energy after the decay is

$$\begin{aligned} K &= \tfrac{1}{2}m_{He}v'^2_{He} + \tfrac{1}{2}m_{Th}v'^2_{Th} \\ &= \tfrac{1}{2}(6.6465 \times 10^{-27}\ \text{kg})(1.423 \times 10^7\ \text{m/s})^2 \\ &\quad + \tfrac{1}{2}(3.8864 \times 10^{-25}\ \text{kg})(-2.432 \times 10^5\ \text{m/s})^2 \\ &= 6.844 \times 10^{-13}\ \text{J} \end{aligned}$$

Clearly, kinetic energy is not conserved in this decay, because the initial kinetic energy of the uranium atom was zero. However, total energy *is conserved*—if we write the total energy as the sum of kinetic energy and nuclear energy, then the total initial energy (kinetic + nuclear) is equal to the total final energy (kinetic + nuclear). Clearly, the gain in kinetic energy occurs as a result of a loss in nuclear energy. This is an example of the type of radioactive decay called alpha decay, which we discuss in more detail in Chapter 12.

Another application of the principle of conservation of energy occurs when a particle moves subject to an external force F. Corresponding to that external force, there is often a potential energy U, defined such that (for one-dimensional motion)

$$F = -\frac{dU}{dx} \tag{1.4}$$

The total energy E is the sum of the kinetic and potential energies:

$$E = K + U \tag{1.5}$$

As the particle moves, K and U may change, but E remains constant. (In Chapter 2, we find that the special theory of relativity gives us a new definition of total energy.)

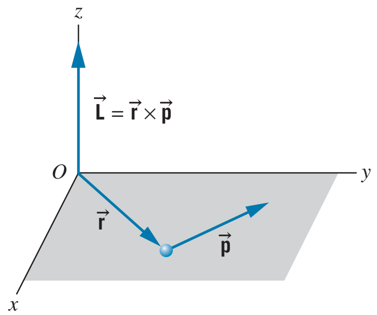


FIGURE 1.3 A particle of mass m, located with respect to the origin O by position vector $\vec{\mathbf{r}}$ and moving with linear momentum $\vec{\mathbf{p}}$, has angular momentum $\vec{\mathbf{L}}$ about O.

When a particle moving with linear momentum $\vec{\mathbf{p}}$ is at a displacement $\vec{\mathbf{r}}$ from the origin O, its *angular momentum* $\vec{\mathbf{L}}$ about the point O is defined (see Figure 1.3) by

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} \tag{1.6}$$

There is a conservation law for angular momentum, just as with linear momentum. In practice, this has many important applications. For example, when a charged particle moves near, and is deflected by, another charged particle, the total angular momentum of the system (the two particles) remains constant if no net external torque acts on the system. If the second particle is so much more massive than the first that its motion is essentially unchanged by the influence of the first particle, the angular momentum of the first particle remains constant (because the second particle acquires no angular momentum). Another application of the conservation of angular momentum occurs when a body such as a comet moves in the gravitational field of Sun—the elliptical shape of the comet's orbit is necessary to conserve angular momentum. In this case, $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$ of the comet must simultaneously change so that $\vec{\mathbf{L}}$ remains constant.

Velocity Addition

Another important aspect of classical physics is the rule for combining velocities. For example, suppose a jet plane is moving at a velocity of $v_{\mathrm{PG}} = 650\ \mathrm{m/s}$, as measured by an observer on the ground. The subscripts on the velocity mean "velocity of the plane relative to the ground." The plane fires a missile in the forward direction; the velocity of the missile relative to the plane is $v_{\mathrm{MP}} = 250\ \mathrm{m/s}$. According to the observer on the ground, the velocity of the missile is $v_{\mathrm{MG}} = v_{\mathrm{MP}} + v_{\mathrm{PG}} = 250\ \mathrm{m/s} + 650\ \mathrm{m/s} = 900\ \mathrm{m/s}$.

We can generalize this rule as follows. Let $\vec{\mathbf{v}}_{AB}$ represent the velocity of A relative to B, and let $\vec{\mathbf{v}}_{BC}$ represent the velocity of B relative to C. Then the velocity of A relative to C is

$$\vec{\mathbf{v}}_{AC} = \vec{\mathbf{v}}_{AB} + \vec{\mathbf{v}}_{BC} \tag{1.7}$$

This equation is written in vector form to allow for the possibility that the velocities might be in different directions; for example, the missile might be fired not in the direction of the plane's velocity but in some other direction. This seems to be a very "common-sense" way of combining velocities, but we will see later in this chapter (and in more detail in Chapter 2) that this common-sense rule can lead to contradictions with observations when we apply it to speeds close to the speed of light.

A common application of this rule (for speeds small compared with the speed of light) occurs in collisions, when we want to analyze conservation of momentum and energy in a frame of reference that is different from the one in which the collision is observed. For example, let's analyze the collision of Example 1.1 in a frame of reference that is moving with the center of mass. Suppose the initial velocity of the He atom defines the positive x direction. The velocity of the center of mass (relative to the laboratory) is then $v_{\mathrm{CL}} = (v_{\mathrm{He}}m_{\mathrm{He}} + v_{\mathrm{N}}m_{\mathrm{N}})/(m_{\mathrm{He}} + m_{\mathrm{N}}) = 3.374 \times 10^5\ \mathrm{m/s}$. We would like to find the initial velocity of the He and N relative to the center of mass. If we start with $v_{HeL} = v_{HeC} + v_{CL}$ and $v_{NL} = v_{NC} + v_{CL}$, then

$$v_{HeC} = v_{HeL} - v_{CL} = 1.518 \times 10^6 \text{ m/s} - 3.374 \times 10^5 \text{ m/s} = 1.181 \times 10^6 \text{ m/s}$$
$$v_{NC} = v_{NL} - v_{CL} = 0 - 3.374 \times 10^5 \text{ m/s} = -0.337 \times 10^6 \text{ m/s}$$

In a similar fashion, we can calculate the final velocities of the He and N. The resulting collision as viewed from this frame of reference is illustrated in Figure 1.4. There is a special symmetry in this view of the collision that is not apparent from the same collision viewed in the laboratory frame of reference (Figure 1.1); each velocity simply changes direction leaving its magnitude unchanged, and the atoms move in opposite directions. The angles in this view of the collision are different from those of Figure 1.1, because the velocity addition in this case applies only to the x components and leaves the y components unchanged, which means that the angles must change.

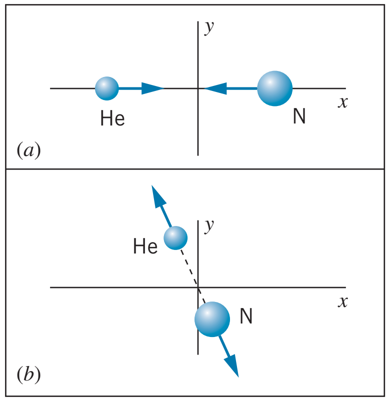


FIGURE 1.4 The collision of Figure 1.1 viewed from a frame of reference moving with the center of mass. (a) Before collision. (b) After collision. In this frame, the two particles always move in opposite directions, and for elastic collisions the magnitude of each particle's velocity is unchanged.

Electricity and Magnetism

The electric field E at a distance r from a point charge q (or at a distance r from the center of a spherically symmetric charge distribution of radius smaller than r) has magnitude

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r} \tag{1.8}$$

The electrostatic force (Coulomb force) exerted by a charged particle q_1 on another charge q_2 has magnitude

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \tag{1.9}$$

The direction of F is along the line joining the particles (Figure 1.5). In the SI system of units, the constant $1/4\pi\varepsilon_0$ has the value

$$\frac{1}{4\pi\varepsilon_0} = 8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

The corresponding potential energy is

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \tag{1.10}$$

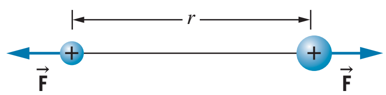


FIGURE 1.5 Two charged particles experience equal and opposite electrostatic forces along the line joining their centers. If the charges have the same sign (both positive or both negative), the force is repulsive; if the signs are different, the force is attractive.

In all equations derived from Eq. 1.8 to 1.10 as starting points, *the quantity* $1/4\pi\varepsilon_0$ *must appear*. In some texts and reference books, you may find electrostatic quantities in which this constant does not appear. In such cases, the centimeter-gram-second (cgs) system has probably been used, in which the constant $1/4\pi\varepsilon_0$ is *defined* to be 1. You should always be very careful in making comparisons of electrostatic quantities from different references and check that the units are identical.

An electrostatic potential difference ΔV can be established by a distribution of charges. The most common example of a potential difference is that between the two terminals of a battery. When a charge q moves through a potential difference ΔV, the change in its electrical potential energy ΔU is

$$\Delta U = q\Delta V \tag{1.11}$$

At the atomic or nuclear level, we usually measure charges in terms of the basic charge of the electron or proton, whose magnitude is $e = 1.602 \times 10^{-19}$ C. If such charges are accelerated through a potential difference ΔV that is a few volts, the resulting loss in potential energy and corresponding gain in kinetic energy will be of the order of 10^{-19} to 10^{-18} J. To avoid working with such small numbers, it is common in the realm of atomic or nuclear physics to measure energies in *electron-volts* (eV), defined to be the energy of a charge equal in magnitude to that of the electron that passes through a potential difference of 1 V:

$$\Delta U = q\Delta V = (1.602 \times 10^{-19}\text{ C})(1\text{ V}) = 1.602 \times 10^{-19}\text{ J}$$

and thus

$$1\text{ eV} = 1.602 \times 10^{-19}\text{ J}$$

Some convenient multiples of the electron-volt are

$$\begin{aligned}\text{keV} &= \text{kilo electron-volt} = 10^3\text{ eV}\\ \text{MeV} &= \text{mega electron-volt} = 10^6\text{ eV}\\ \text{GeV} &= \text{giga electron-volt} = 10^9\text{ eV}\end{aligned}$$

(In some older works you may find reference to the BeV, for billion electron-volts; this is a source of confusion, for in the United States a billion is 10^9 while in Europe a billion is 10^{12}.)

Often we wish to find the potential energy of two basic charges separated by typical atomic or nuclear dimensions, and we wish to have the result expressed in electron-volts. Here is a convenient way of doing this. First we express the quantity $e^2/4\pi\varepsilon_0$ in a more convenient form:

$$\begin{aligned}\frac{e^2}{4\pi\varepsilon_0} &= (8.988 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.602 \times 10^{-19}\text{ C})^2 = 2.307 \times 10^{-28}\text{ N}\cdot\text{m}^2\\ &= (2.307 \times 10^{-28}\text{ N}\cdot\text{m}^2)\left(\frac{1}{1.602 \times 10^{-19}\text{ J/eV}}\right)\left(\frac{10^9\text{ nm}}{\text{m}}\right)\\ &= 1.440\text{ eV}\cdot\text{nm}\end{aligned}$$

With this useful combination of constants, it becomes very easy to calculate electrostatic potential energies. For two electrons separated by a typical atomic dimension of 1.00 nm, Eq. 1.10 gives

$$U = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{r} = (1.440\text{ eV}\cdot\text{nm})\left(\frac{1}{1.00\text{ nm}}\right) = 1.44\text{ eV}$$

For calculations at the nuclear level, the femtometer is a more convenient unit of distance and MeV is a more appropriate energy unit:

$$\frac{e^2}{4\pi\varepsilon_0} = (1.440\text{ eV}\cdot\text{nm})\left(\frac{1\text{ m}}{10^9\text{ nm}}\right)\left(\frac{10^{15}\text{ fm}}{1\text{ m}}\right)\left(\frac{1\text{ MeV}}{10^6\text{ eV}}\right) = 1.440\text{ MeV}\cdot\text{fm}$$

It is remarkable (and convenient to remember) that the quantity $e^2/4\pi\varepsilon_0$ has the same value of 1.440 whether we use typical atomic energies and sizes (eV · nm) or typical nuclear energies and sizes (MeV · fm).

A magnetic field $\vec{\mathbf{B}}$ can be produced by an electric current i. For example, the magnitude of the magnetic field at the center of a circular current loop of radius r is (see Figure 1.6*a*)

$$B = \frac{\mu_0 i}{2r} \tag{1.12}$$

The SI unit for magnetic field is the tesla (T), which is equivalent to a newton per ampere-meter. The constant μ_0 is

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N \cdot s^2/C^2}$$

Be sure to remember that i is in the direction of the conventional (*positive*) current, opposite to the actual direction of travel of the negatively charged electrons that typically produce the current in metallic wires. The direction of $\vec{\mathbf{B}}$ is chosen according to the right-hand rule: if you hold the wire in the right hand with the thumb pointing in the direction of the current, the fingers point in the direction of the magnetic field.

It is often convenient to define the *magnetic moment* $\vec{\boldsymbol{\mu}}$ of a current loop:

$$|\vec{\boldsymbol{\mu}}| = iA \tag{1.13}$$

where A is the geometrical area enclosed by the loop. The direction of $\vec{\boldsymbol{\mu}}$ is perpendicular to the plane of the loop, according to the right-hand rule.

When a current loop is placed in a uniform *external* magnetic field $\vec{\mathbf{B}}_{\mathrm{ext}}$ (as in Figure 1.6*b*), there is a torque $\vec{\tau}$ on the loop that tends to line up $\vec{\boldsymbol{\mu}}$ with $\vec{\mathbf{B}}_{\mathrm{ext}}$:

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}_{\mathrm{ext}} \tag{1.14}$$

Another way to describe this interaction is to assign a potential energy to the magnetic moment $\vec{\boldsymbol{\mu}}$ in the external field $\vec{\mathbf{B}}_{\mathrm{ext}}$:

$$U = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}_{\mathrm{ext}} \tag{1.15}$$

When the field $\vec{\mathbf{B}}_{\mathrm{ext}}$ is applied, $\vec{\boldsymbol{\mu}}$ rotates so that its energy tends to a minimum value, which occurs when $\vec{\boldsymbol{\mu}}$ and $\vec{\mathbf{B}}_{\mathrm{ext}}$ are parallel.

It is important for us to understand the properties of magnetic moments, because particles such as electrons or protons have magnetic moments. Although we don't imagine these particles to be tiny current loops, their magnetic moments do obey Eqs. 1.14 and 1.15.

A particularly important aspect of electromagnetism is *electromagnetic waves*. In Chapter 3, we discuss some properties of these waves in more detail. Electromagnetic waves travel in free space with speed c (the speed of light), which is related to the electromagnetic constants ε_0 and μ_0:

$$c = (\varepsilon_0 \mu_0)^{-1/2} \tag{1.16}$$

The speed of light has the exact value of $c = 299{,}792{,}458$ m/s.

Electromagnetic waves have a frequency f and wavelength λ, which are related by

$$c = \lambda f \tag{1.17}$$

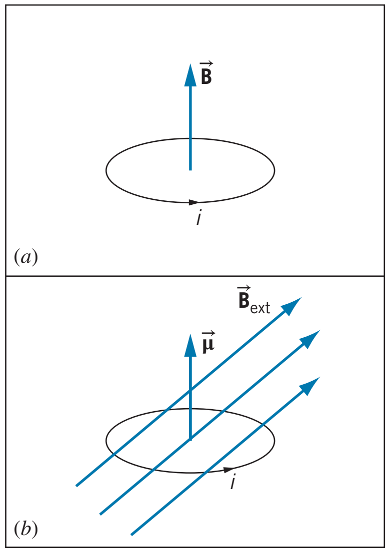


FIGURE 1.6 (*a*) A circular current loop produces a magnetic field $\vec{\mathbf{B}}$ at its center. (*b*) A current loop with magnetic moment $\vec{\boldsymbol{\mu}}$ in an external magnetic field $\vec{\mathbf{B}}_{\mathrm{ext}}$. The field exerts a torque on the loop that will tend to rotate it so that $\vec{\boldsymbol{\mu}}$ lines up with $\vec{\mathbf{B}}_{\mathrm{ext}}$.